Determine size of footing

Column subjected to axial column load only

Since factor of safety is included in determining allowable soil bearing capacity, there is no need to add addition factor of safety in determine the footing sizes. But, since the bottom of footing is at a depth below ground surface, the weight of soil and footing above the bearing area should be subtracted from the allowable soil capacity. The required footing area is column load divided by the net allowable soil bearing capacity.

A = P / Qa-net [2.1]

Where
A: required footing area.
P: Axial column load
Qa net = net allowable soil bearing capacity.

The weigh of footing and the soil above should be heavy enough to offset the uplift forces from wind or seismic.

Wt $\geq$ U * F.S. [2.2]

Where:
Wt: Total weight of footing
U: uplift force
F.S.: factor of safety.
This situation usually occurs at column at building bracing location. The factor of safety for uplift force in most of building codes is 1.5.

Example 1: Determine footing sizes for axial loads and uplift.

Given:

- Column loads:
- Live load: 25 kips
- Dead load: 25 kips
- Uplift = 20 kips
- Factor of safety for uplift = 1.5
- Footing information:
- Top of footing at 1 ft below ground surface, unit weigh weight of soil: 100 lbs/ft^3.
- Allowable soil bearing capacity = 3000 psf
- Unit weight of concrete: 150 lbs/ft^3.

Requirement: Determine footing sizes for axial loads and uplift.

Solution:

1. Total column service load = 25+25=50 kips
2. Assume a footing depth of 1 ft,
3. Net allowable soil bearing capacity = 3000-150*1-100*1=2750 psf
4. Required footing area = 50*1000/2750=18.2 ft^2.
5. Try 4'6"x4'6" footing, footing area = 20.2 ft^2.
6. Required weight of footing to offset uplift = 20*1.5=30 kips
7. Weight of footing above footing = 100*4.5*4.5/1000=2.0 kips
8. Required weight of footing = 30-25-2=3 kips
9. Required volume of footing = 3/0.15=20 ft^3.
10. Required depth of footing = 20/20.2 =1 ft
11. Use 4'6"x4'6"x1' footing.

Column subjected to both axial column load and moment or eccentric loading

Columns at the base of a moment revisiting frame are often subjected to moment in addition to axial load. Columns that at edge of buildings often have to be designed with eccentricity due to limitation of property line. The bearing pressure at the bottom of footing will distribute in trapezoidal or triangular shape. The footing has to be sized so that maximum footing pressure does not exceed allowable soil bearing capacity.

Eccentricity is within 1/6 width of footing

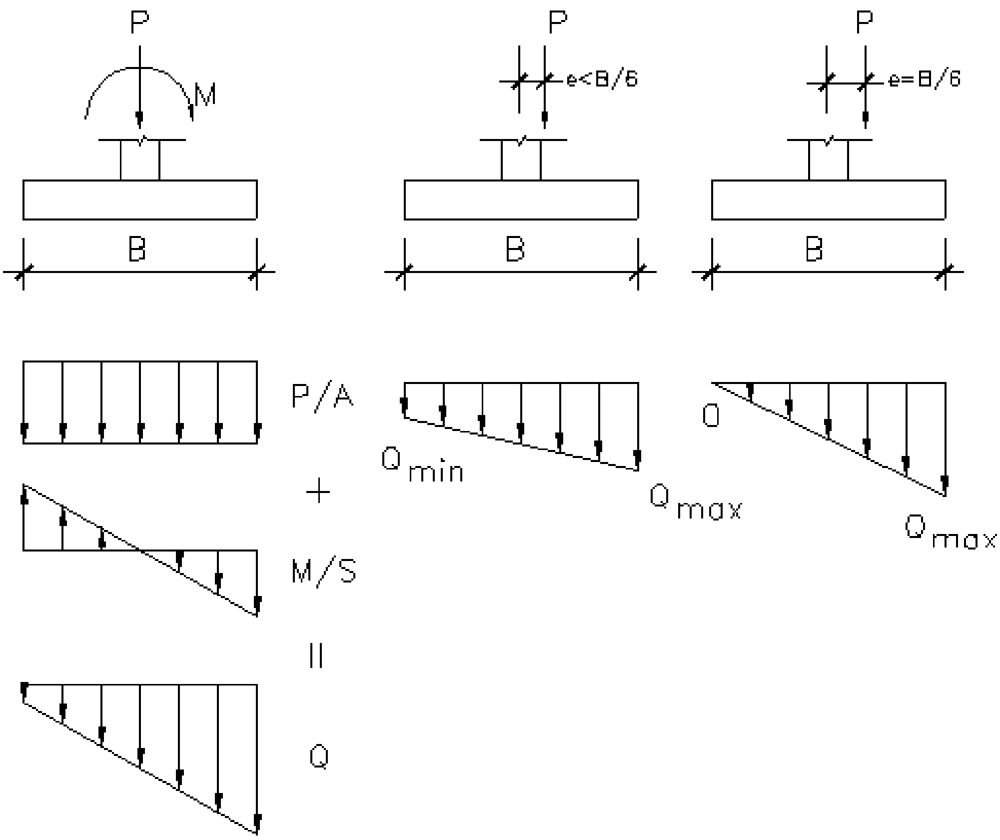


Figure 2.1 Footing pressures with eccentricity not more than 1/6 footing width

When eccentricity is less than 1/6 width of footing, footing pressure under the footing is distributed in trapezoidal shape. When eccentricity equals to 1/6 width of footing, footing pressure distributes triangularly with zero pressure at one end of the footing.

The soil bearing capacity can be calculated as

$$Q = P / A \pm M / S \qquad [2.3]$$

P: Axial column Load
A: footing area
M = P*e, column moment in the x direction, e is eccentricity in x direction.
S = $LB^2/6$ section modulus of footing area in x direction

For a rectangular footing, the equation can be written as

$$Q = P / A \pm M / S$$
$$= P/(BL) \pm P*e/(LB^2/6)$$
$$= (P/A) [1 \pm e*B/6] \quad [2.4]$$

L, B are length and width of footing.

When footing is subjected to moments or eccentricities in both direction, the equations become

$$Q = P / A \pm Mx / Sx \pm My / Sy \quad [2.5]$$

Or

$$Q = (P/A) [1 \pm e_x*B/6 \pm e_y*L/6] \quad [2.6]$$

Example 2: Determine maximum and minimum footing pressure for footing with eccentricity < B/6.

Given:

- Column loads:
- Live load: 25 kips
- Dead load: 25 kips
- Live load moment = 20 ft-kips
- Dead load moment = 20 kips
- Footing information:
- Footing sizes = 6 ft x 6 ft

Requirement: Determine maximum and minimum footing pressure.

Solution:

1. Total axial load = 25+25=50 kips
2. Total column moment = 20+20=40 ft-kips
3. Eccentricity = 40/50=0.8 ft < B/6 = 1 ft
4. Maximum footing pressure = [50,000/(6x6)][1+0.8*6/6] = 2500 psf
5. Minimum footing pressure = [50,000/(6x6)][1-0.8*6/6] = 277 psf

Eccentricity exceeds 1/6 width of footing

When eccentricity exceeds 1/6 width of footing, soil pressure under pressure distributes in a triangular shape with a portion of the footing have zero pressure. The resultant of footing pressure, R coincides with column load, P as shown below. Since the center of the resultant is at 1/3 length of the triangle, the length of the bearing area is three times of the distance from the center of the column load to the edge of footing.

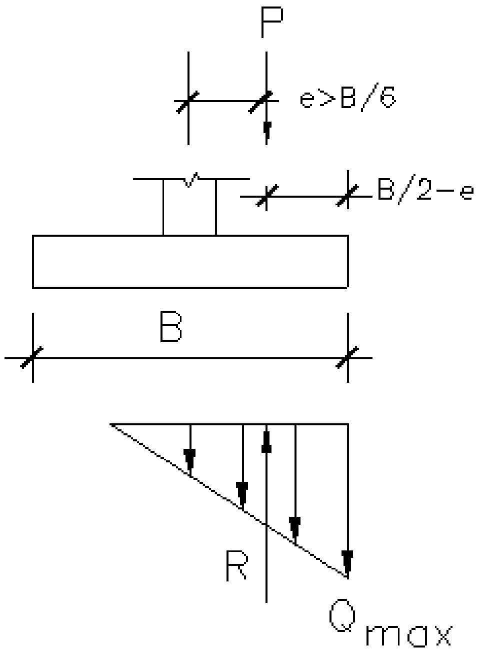


Figure 2.2 Footing pressure with eccentricity greater than 1/6 footing width

Therefore,
P = Qmax [3(B/2-2)L/2]
Then,
Qmax = 2P/[3(B/2-e)L] [2.7]

Example 3: Determine maximum footing pressure for footing with eccentricity > B/6
Given:
- Column loads:
- Live load: 25 kips
- Dead load: 25 kips
- Live load moment = 30 ft-kips
- Dead load moment = 30 kips
- Footing information:
- Footing sizes = 6 ft x 6 ft

Requirement: Determine maximum and minimum footing pressure.
Solution:

1. Total axial load = 25+25=50 kips
2. Total column moment = 30+30=60 ft-kips
3. Eccentricity = 60/50=1.2 ft > B/6 = 1 ft
4. Maximum footing pressure = 2*50,000/[3*(6/2-1.2)*6] = 3086 psf

Determine depth of spread footings for shear

This portion of reinforced concrete design of spreading footing follows the requirement of ACI code 318-99. Factored loads should be used instead of service load. Factored footing pressure is used to determine footing depth and reinforcement.

The topics include:

Determine depth of footing for punching shear and direct shear

The depth of footing usually governs by punching shear and direct shear because shear reinforcement is normally not used. For a square footing, punching shear usually governs the design. For a rectangular footing, direct shear may be more critical.

Punching shear (Two-way shear)

The critical section of punching shear is located at one half effective distance from the faces of column. Shear strength of concrete should be larger than factored shear stress at critical section as

$$\phi v_c \geq v_u$$

The punching shear strength ϕv_c in ACI is

$$\phi v_c = \phi\left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'} \leq \phi 4\sqrt{f_c'} \qquad [2.8]$$

where = 0.85 for shear, $_c$ is the ratio of long to short sides of column, and f_c' is the compressive strength of concrete.

The punching shear stress is factored shear force at the critical section divided by the perimeter of the critical section and the effective depth of the footing. The factored shear force at the critical section is factored column load minus factored footing pressure under the critical section. It can be calculated as

$$v_u = \frac{Q_u \times [BL - (b+d)(c+d)]}{2(b+d)(c+d)d} \qquad [2.9]$$

where

v_u is punching shear stress,
Q_u is factor footing pressure,
B, L are width and length of footing,
b, c are width and length of square column
d is effective depth of the footing.

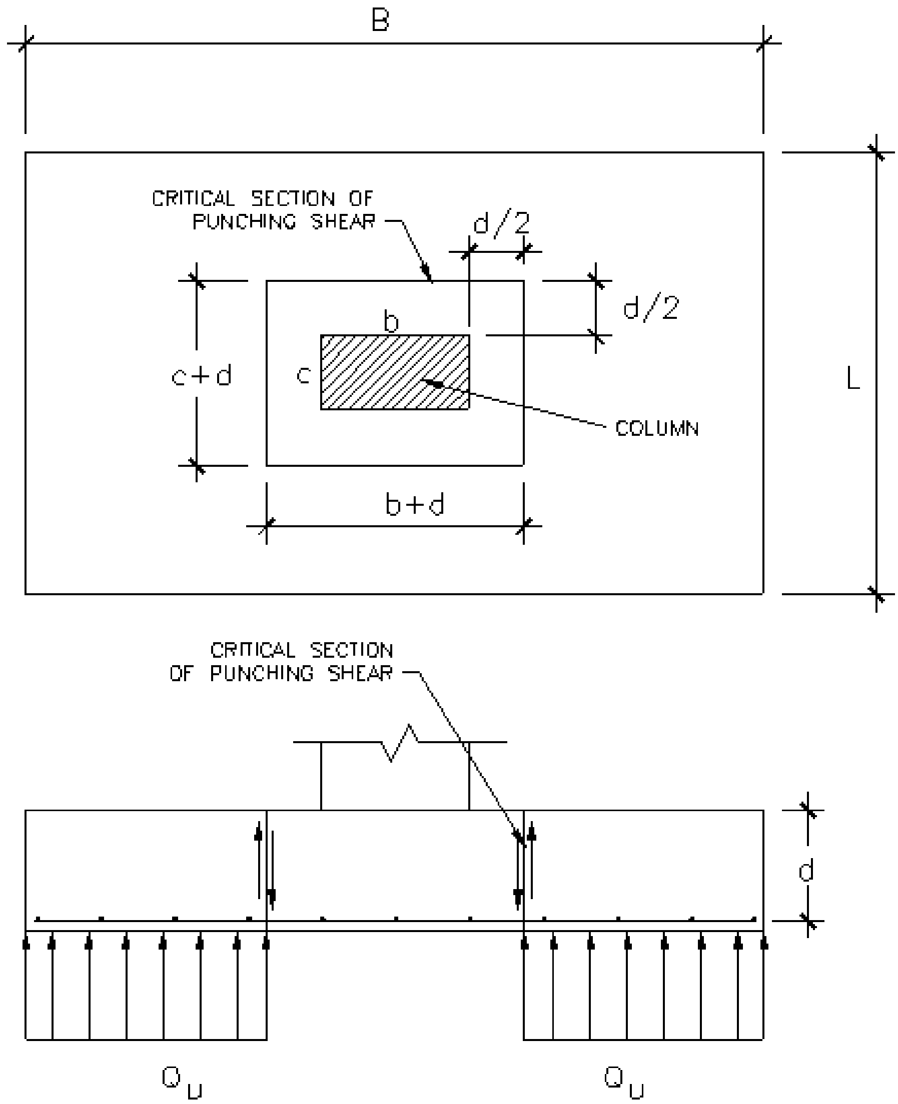


Figure 2.3 Critical sections of punching shear

Director shear (One-way shear)

The critical section of direct shear is at one effective distance from the face of column. The direct shear strength of specified in ACI is

$$\phi v_c = \phi 2\sqrt{f'_c} \qquad [2.10]$$

The factored shear stress at the critical section is the factored shear force divided by the width and depth of the footing at the critical section. The factored shear force at the critical section is the area from the critical section to the edge of the footing multiply average factored footing pressure in the area.

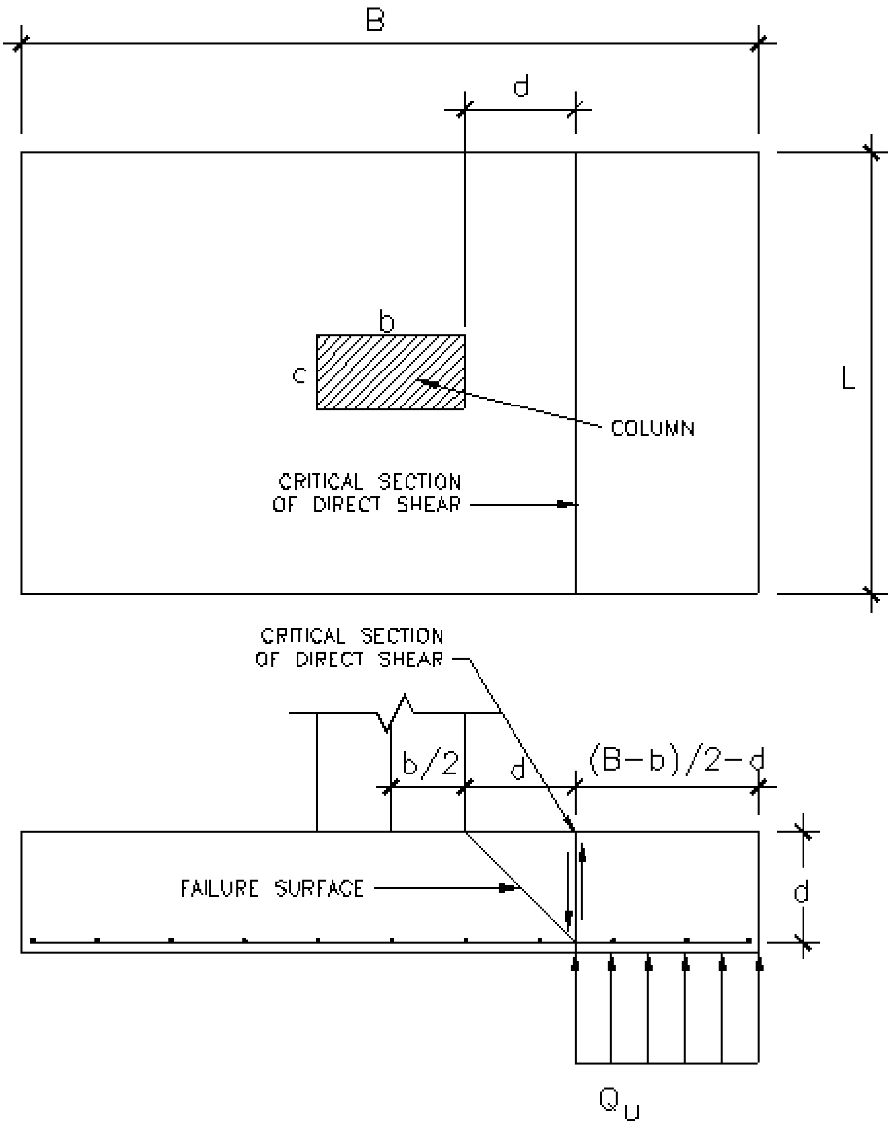


Figure 2.4. Critical section of direct shear

Example 4: Check footing depth for punching shear and direct shear for footing subjected to axial column load only

Given:

- Column loads:
- Live load: 25 kips
- Dead load: 25 kips
- Footing and column information:
- Footing sizes = 4 ft x 4 ft x 1ft
- Column size: 1 ft x 1 ft
- Concrete strength at 28 day = 3000 psi

Requirement: Check if the footing depth is adequate for punching shear and direct shear.

Solution:

Factored column load = 1.4*25+1.7*25= 77.5 kips

Factored footing pressure = 77.5/16=4.84 ksf

Effective footing depth = 12”-3” (cover) – 0.5” (assumed diameter of #4 bar) = 8.5”
Check punching shear:
Area under critical section = [(12”+8.5”)/12]^2=2.92 ft^2.
Factored shear force at critical section = 77.5-4.84*2.92=63.4 kips
Perimeter of critical section = 4 (12” + 8.5”) = 82”
Factor shear stress at critical section = 63,400/(82*8.5) = 90.9 psi
Punching shear strength of concrete = 0.85*(4√3000) = 186 psi > 90.9 psi O.K.
Check direct shear:
Distance from critical section to edge of footing = 4/2-1/2-8.5/12=0.83’
Factored shear force = 4.84*0.83*4=16.1 kips
Factored shear stress = 16,100/(4*12*8.5) = 39.5 psi
Director shear stress of concrete = 0.85*(2√3000) = 93 psi >39.5 psi O.K.

Example 5: Check footing depth for punching shear and direct shear for footing subjected to axial column load and moment

Given:

- Column loads:
- Live load: 25 kips
- Dead load: 25 kips
- Live load moment = 25 ft-kips
- Dead load moment = 25 ft-kips
- Footing and column information:
- Footing sizes = 6 ft x 4 ft x 1ft
- Column size: 1 ft x 1 ft
- Concrete strength at 28 day = 3000 psi

Requirement: Check if the footing depth is adequate for punching shear and direct shear.
Solution:
Factored column load = 1.4*25+1.7*25= 77.5 kips
Factored column moment = 1.4*25+1.7*25=77.5 ft-kips
Eccentricity of factored column load = 77.5/77.5 = 1 ft
Maximum footing pressure = [77.5/(4x6)][1+1*6/6]=6.46 ksf
Minimum footing pressure = [77.5/(4x6)][1-1*6/6]=0 ksf
Effective footing depth = 12”-3” (cover) – 0.5” (assumed diameter of #4 bar) = 8.5”
Check punching shear:
Average factored footing pressure = (6.46+0)/2=3.23 ksf
Area under critical section = [(12”+8.5”)/12]^2=2.92 ft^2.
Factored shear force at critical section = 77.5-3.23*2.92= 68.1 kips
Perimeter of critical section = 4 (12” + 8.5”) = 82”
Factor shear stress at critical section = 68,100/(82*8.5) = 97.8 psi
Punching shear strength of concrete = 0.85*(4√3000) = 186 psi > 90.9 psi O.K.
Check direct shear:
Distance from critical section to edge of footing with maximum footing pressure
= 6/2-1/2-8.5/12=1.83’
Distance from critical section to edge of footing with maximum footing pressure
= 6-1.83=4.17’
Factor footing pressure at the location of critical section=6.46*4.17/6=4.49 ksf
Factored shear force = [(6.46+4.49)/2]*1.83*4=40.1 kips
Factored shear stress = 40,100/(4*12*8.5) = 98.3 psi
Director shear stress of concrete = 0.85*(2√3000) = 93 psi <98.3 psi N.G. Need to increase footing depth.

Design reinforcement for spread footings

This portion of reinforced concrete design of spreading footing follows the requirement of ACI code 318-99. Factored loads should be used instead of service load. Factored footing pressure is used to determine footing reinforcement.

The topics include:

- Moment calculation
- Calculating Reinforcement
- Minimum and maximum reinforcements
- Example 6: Determine footing reinforcement for footing subjected to axial column load

- Placing reinforcements.
- Design column dowels
- Bearing strength of concrete at base of column
- Reinforcement required at the base of column
- Length of dowel for compression
- Example 7: Design of column dowel

Design footing reinforcements

Moment calculation

The footing needs to be reinforced for the bending moment producing from upward footing pressure. According to ACI code, the critical section is at the face of column. The factored moment at the critical section can be calculated as

$$M_u = Q_u * l^2/2 \qquad [2.11]$$

Where

Q_u is factored footing pressure

l is the distance from the face of column to the edge of footing.

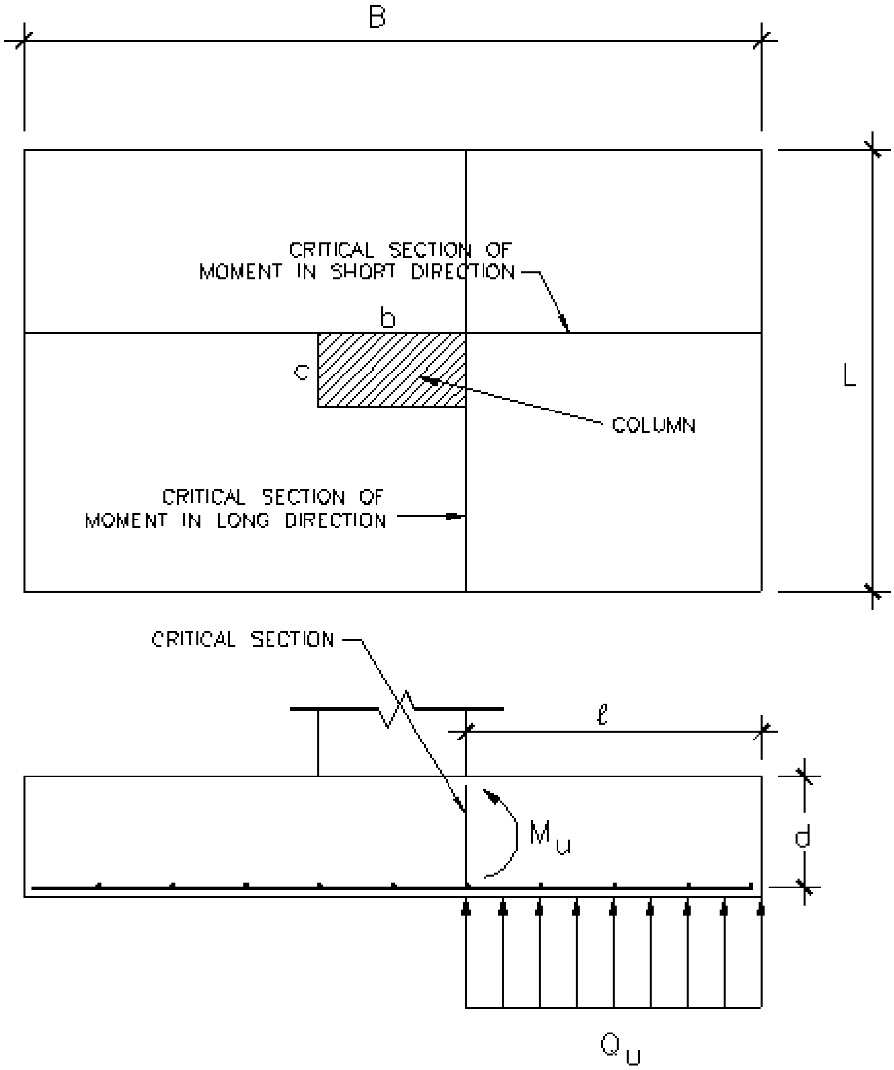


Figure 2.5 Critical moment section of a footing

Calculating Reinforcement

The footing reinforcements are designed based on ACI strength design method. At ultimate stress situation, the concrete at top portion is subjected to compression. The compressive stresses distribute uniformly over a depth a. The resultant of compressive stress, C is located at a distance, a/2, from the top surface. Tensile force is taken by rebars at an effective distance, d, from the top surface.

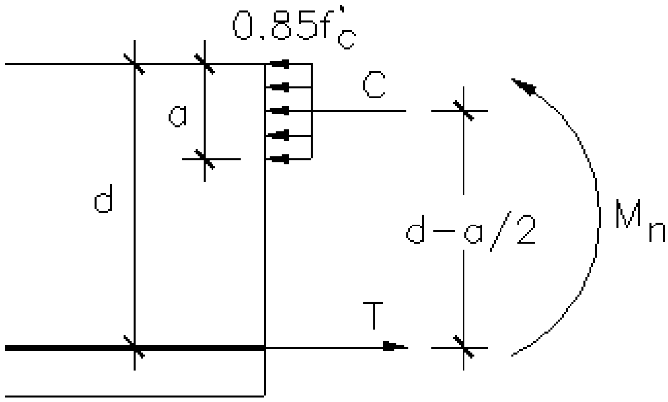


Figure 2.6 Tensile and compressive forces and stresses on a footing secti0n

According to ACI code, the ultimate compressive is $0.85f'_c$, where f'_c is compressive strength of concrete. Therefore, the compressive stress, C = 0.85 f'c a b, where b is the width of the footing in calculation. By equilibrium, the tensile force is equal to the compression resultant,

$T = C = 0.85f'_c\ a\ b$ [2.12]

Therefore,

$$a = \frac{T}{0.85 f'_c b}$$ [2.13]

The nominal moment strength of the section,

Mn = T (d-a/2) [2.14]

ACI code requires that the factored moment,

Mu ≤ ϕ Mn

Where, ϕ = 0.9, is the strength reduction factor for beam design. Then, the tensile force,

$$T = \frac{M_u}{\phi(d - \frac{a}{2})}$$ [2.15]

and, the area of reinforcement is

$$A_s = \frac{T}{F_y}$$ [2.16]

where Fy is the yield strength of reinforcing steel.

There are many ways to determine reinforcements. One simple method is using a trial and error method by assuming the depth of compression block, a. The steps are as follows

1. Assume a depth of the stress block, a.
2. Calculate tensile force, T using equation [2.16]
3. Calculate new depth of the stress block, a, using equation [2.13]
4. If the new depth, a, is not close to the assumed, a, in step 1, repeat step 2 and 3 with new depth, a.
5. If the new depth, a, is close to the assume, a, calculate area of reinforcement using equation [2.16]

Minimum and maximum reinforcements

ACI code requires the minimum reinforcement ratio,

$$\rho = \frac{As}{Bd} \geq \rho_{min} \frac{200}{f_y} \qquad [2.17]$$

Where, As is the area of reinforcements. In addition, it also said that the minimum reinforcement does not need to be more than 4/3 of the calculated value,

$$\rho_{min} \leq \frac{4}{3}\rho \qquad [2.18]$$

Example 6: Determine footing reinforcement for footing subjected to axial column load

Given:

- Column loads:
- Live load: 25 kips
- Dead load: 25 kips
- Footing and column information:
- Footing sizes = 4 ft 6 in. x 4 ft 6 in. x 1ft
- Column size: 1 ft x 1 ft
- Concrete strength at 28 day = 3000 psi
- Yield strength of rebars = 60 ksi

Requirement: Calculate footing reinforcements

Solution:

1. Calculate factored column load,
2. Pu = 1.4*25+1.7*25=77.5 kips
3. Factored footing pressure = 77.5/(4.5*4.5) =3.827 ksf
4. Distance from critical section to edge of footing = (4.5-1)/2=1.75'
5. Factored moment at critical section = (3.827)*1.75^2/2=5.86 ft-kips/ft
6. Effective depth = 12"-3" (cover)-0.5" (rebar size) = 8.5"
7. Assume a = 1"
8. T = (5.86*12)/[0.85*(8.5-1/2)]=10.3 kips/ft
9. Check a = 10.3/[0.85*3*12] =0.34"
10. Assume a = 0.34"
11. T = (5.86*12)/[0.85*(8.5-0.34/20]=9.9 kips
12. Check a = 9.9/[0.85*3*12]=0.32" (close enough)
13. As =9.9/60=0.165 in^2/ft
14. The reinforcement ratio,ρ = 0.165/[8.5*12]=0.0016
15. Less than ρ_{min} = 200/ 60=0.0033 or ρ_{min} = (4/3) 0.00162=0.00216
16. For a footing width of 4'6", As = 0.216*8.5*4.5*12= 0.99in^2.
17. Use 5#4 in both direction, As = 5*0.2=1.0 in^2.

Placing reinforcements.

Reinforcements should be placed at the tension side at the bottom of the footing.

For a square footing, rebars are placed uniformly in both directions. ACI code requires that the rebars be placed not more than 18 inch apart.

For a rectangular footing, rebars in the long direction are placed uniformly but not the short direction. ACI code requires a certain portion of reinforcements in short direction to be placed within a band equal to the width of footing in the short direction. The distribution ratio is calculated based on the aspect ratio of footing as

$$r = \frac{2}{\beta + 1} \qquad [2.19]$$

where is the ratio of length to short side.

Design column dowels

Dower rebars that go from the bottom of footing into the footing need the meet the following requirements:

1. Transfer vertical column forces when column load exceeds the compressive strength of concrete.
2. Transfer moment at column base
3. Meet minimum reinforcement in ACI code
4. Meet splice requirement for column reinforcement.

Bearing strength of concrete at base of column

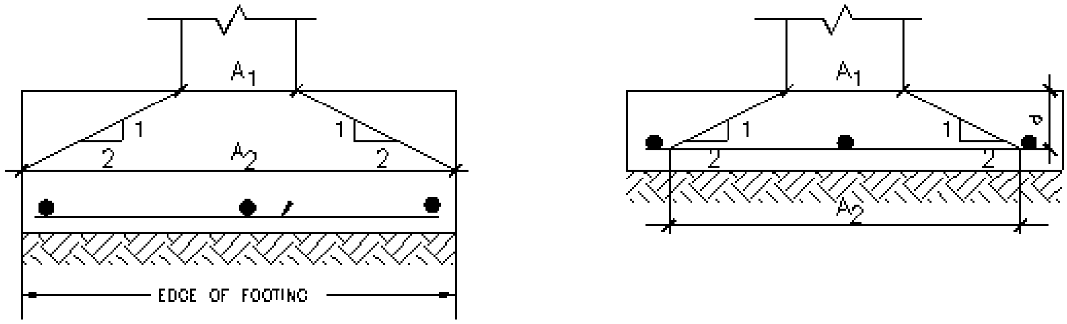
The bearing strength of column at the column base

$\phi Pc = 0.7*0.85fc'Ag$

Where Ag is the gross section area of column
The bearing strength of footing at the column base is

$\phi Pc = 0.7*0.85fc'\alpha Ag$ $\alpha = \sqrt{A_2/A_1} \leq 2.$

Edge length of A_1, A_2 are shown in the figure below.



Reinforcement required at the base of column

Where there is no moment at the column base, the area reinforcement through column base can be calculated as

$As = (Pu - \phi Pc) / fy$ [2.20]

Pu is factored column load. When $Pu < \phi Pc$, ACI code requires that the minimum reinforcement for dowel through column base is 0.005Ag. Ag is the cross section area of column. The diameter of the dowel should not exceeds the longitudinal reinforcement of column by 0.15 in.

When the column base is subjected to both axial loads and moments, the column dowel needs to be designed to resist column moment. The design procedure is the same as design of beam-columns.

Length of dowel for compression

The length of dowel that below the column base need to meet minimum development length of ACI code.

Basic development length for compression member is the larger of

$L_{db} = 0.02\ (fyd_b/\sqrt{fc'})$ [2.21]

$L_{db} = 0.0003\ fyd_b$ [2.22]

Where d_b is the diameter of rebar

The length of dowel is modified by the area of reinforcement as

$Ld = (As\ required / As\ provided)(L_{db})$ [2.23]

The length of dowel that projects above the footing needs to meet the compression splice requirement of column reinforcement.

When, $fy \leq 60{,}000$ psi, $Lap = 0.0005\ fyd_b$, > Ld or 12" [2.24]

When, fy > 60,000 psi, Lap = $(0.0009\ fy - 24)d_b$ > Ld or 12” [2.25]

When column base is subject to moment and the rebars are in tension, length of splice and anchor should be designed based on tension requirement.

Example 7: Design of column dowel

Given:

- Column loads:
- Live load: 20 kips
- Dead load: 40 kips
- Footing and column information:
- Footing sizes = 4 ft x 4 ft x 1ft
- Column size: 1 ft x 1 ft
- Concrete strength at 28 day for footing = 3000 psi
- Concrete strength at 28 day for column = 4000 psi
- Yield strength of rebars = 60 ksi
- Column reinforcement: 4#6
- Footing reinforcement: 4#4 each way.

Requirement: Design column dowels including sizes and length

Solution:

Determine number and size of rebar

1. Factored column load = 1.4*40+1.7*20=90 kips
2. Bearing strength of column = 0.7*0.85*4*12*12=342 kips >90 kips
3. A_1 = 12*12=144 in^2.
4. Effective depth = 12-3-1=8 in.
5. Edge length of A_2 = 12+8*2*2 = 44” < 48”
6. Area of A_2 = 44*44=1936 in^2.
7. The ratio, $\alpha = \sqrt{A_2/A_1} = \sqrt{1936/144} = 3.7$ >2 Use 2.
8. The bearing strength of footing at column base = 0.7*0.85*2*3*144=514 kips >90 kips
9. Use minimum reinforcement, As = 0.005*144=0.72 in^2
10. Use 4#4, As = 0.8 in^2

Length of dowel in footing:

1. Basic development for #4 bars:
2. $L_{db} = 0.02\ (fyd_b/\sqrt{fc'}) = 0.02(60,000*0.5/\sqrt{3000}) = 11”$
3. L_{db} =0.0003 fyd_b=0.0003*60000*0.5= 9 in.
4. Required development length, L_d = 11”*(0.72/0.8) = 9.9”
5. Since the distance from the top of footing to the #4 bottom reinforcement is 7.5”, use dowel with 90 degree hook, 7.5 in vertical and 2.5 “ turn.

Splice length in column:

1. For #6 bars in column,
2. Lap = 0.0005 fyd_b = 0.0005*60000*0.75=22.5 in
3. Ld = $0.02*60000*0.75/\sqrt{40000}$=14 in.
4. Ld = 0.0003*60000*0.75=13.5 in.
5. Use Lap = 22.5 in.

Design of square footings

The procedure for designing a square footing is as follows:

Service load design:

1. Determine size of footing.

Reinforced concreter design:

2. Determine depth of footing for punching shear and direct shear
3. Determine footing reinforcement for bending moment.
4. Determine column dowel to transfer column load.

Example 8: Design of a square footing

Given:

- Column loads:
- Live load: 100 kips

- Dead load: 100 kips
- Footing uplift: 0 kips
- Column size: 1 ft. x 1 ft.
- Soil information:
- Allowable soil bearing capacity: 4000 psf
- Soil cover above footing: 1 ft
- Unit weight of soil: 120 pcf
- Materials used:
- Concrete strength at 28 day = 3000 psi
- Yield strength of rebars = 60 ksi

Requirement: Determine size, depth, and reinforcement for a square footing.

Solution:

Service load design:

1. Determine footing sizes:

1. Assume a footing depth of 18",net soil bearing capacity ,
2. Q_{net} = 4000 – 150*18/12-120*1 = 3655 psf
3. Required footing are, A = (100+100) (1000) / 3655 = 54.7 ft^2
4. Use 7'-6" by 7'-6" square footing. The footing area is 56.3 ft^2.

Reinforced concrete design:

2. Determine footing depth

The factored footing pressure can be calculated as

Q_u = (1.4 x 120 + 1.7 x 80) / 56.3 = 5.5 psf

a. Check punching shear

1. Assume the reinforcements are #6 bars, the effective depth
2. d = 18" - 3" (cover) - 0.75" (one bar size) = 14.3 " = 1.2'
3. The punch shear stress can be calculated as
4. $$v_u = \frac{5.5\times\left[7.5^2-(1+1.2)^2\right](1000)}{4\times1.2\times(1+1.2)(144)} = 186\ psi$$
5. The shear strength of concrete is
6. ϕv_c = 0.85 x 4 x 3000 = 186 psi O.K.

b. Check direct shear:

1. The distance from the critical section of direct shear to the edge of the footing,
2. l = (7.5)(12)/2 - 1/2 – 1.2 = 24.75" = 2.05'
3. The direct shear stress is
4. v_u = (5.5)(1000)(2.05) / (12)(14.3) = 65.7 psi per foot width of footing.
5. The shear strength of concrete for direct shear is
6. ϕv_c = 0.85 x 2 x 3000 = 93 psi > 65.7 psi O.K.

3. Determine footing reinforcement

1. The distance from face of column to the edge of the footing is
2. l = 7.5/2 – 1/2 = 3.25'
3. The factored moment at the face of the column is
4. M_u = (5.5)(3.25)^2/2 = 29 k-ft. per foot width of footing
5. Use trail method for reinforcement design
6. Assume a = 0.9".
7. $$T = \frac{M_u}{\phi(d-\frac{a}{2})} = \frac{(29)(12)}{(0.9)(14.3-\frac{0.9}{2})} = 27.9\ kips$$
8. Calculate new **a**,
9. $$a = \frac{T}{0.85f'_c b} = \frac{27.9}{(0.85)(3)(12)} = 0.91\ in.$$ ≈ 0.9”
10. $$A_s = \frac{T}{f_y} = \frac{29}{60} = 0.48\ in^2$$ at one foot section.
11. The reinforcement ratio is
12. $$\rho = \frac{A_s}{bd} = \frac{0.48}{(12)(14.3)} = 0.0028$$