Terzaghi's Bearing Capacity Equations

- Bearing capacity equation
- Bearing capacity factors
- Bearing capacity Chart
- Example 1: Strip footing on cohesionless soil
- Example 2: Square footing on clay soil
- Example 3: Circular footing on sandy clay

Terzaghi's bearing capacity theory

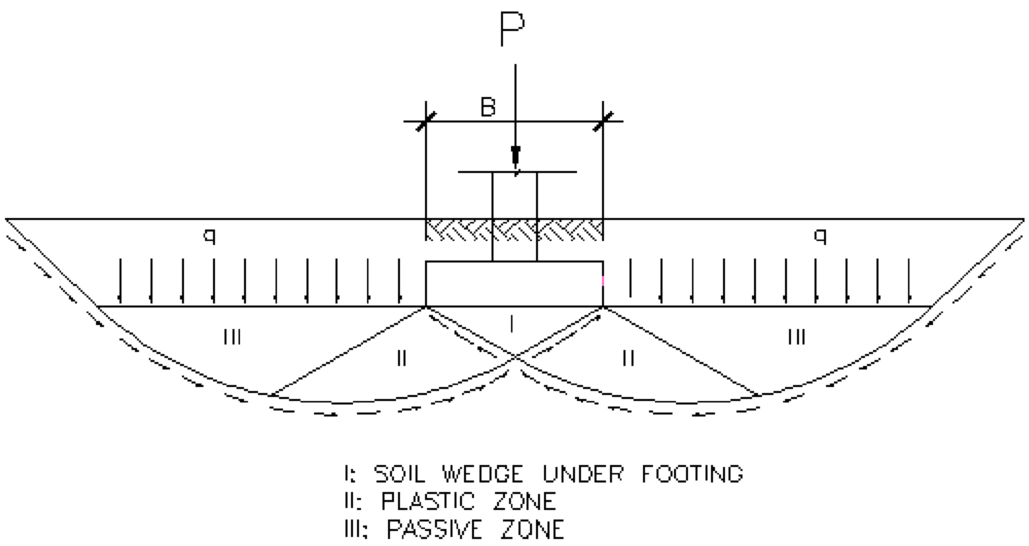


Figure 1.1: Shear stresses based on Terzaghi's soil bearing capacity theory

Based on Terzaghi's bearing capacity theory, column load P is resisted by shear stresses at edges of three zones under the footing and the overburden pressure, q (=γD) above the footing. The first term in the equation is related to cohesion of the soil. The second term is related to the depth of the footing and overburden pressure. The third term is related to the width of the footing and the length of shear stress area. The bearing capacity factors, Nc, Nq, Nγ, are function of internal friction angle, ϕ.

Terzaghi's Bearing capacity equations:

Strip footings:

$$Qu = c\,Nc + \gamma\,D\,Nq + 0.5\,\gamma\,B\,N\gamma \quad [1.1]$$

Square footings:

$$Qu = 1.3\,c\,Nc + \gamma\,D\,Nq + 0.4\,\gamma\,B\,N\gamma \quad [1.2]$$

Circular footings:

$$Qu = 1.3\,c\,Nc + \gamma\,D\,Nq + 0.3\,\gamma\,B\,N\gamma \quad [1.3]$$

Where:
C: Cohesion of soil, γ : unit weight of soil, D: depth of footing, B: width of footing
Nc, Nq, Nr: Terzaghi's bearing capacity factors depend on soil friction angle, ϕ.

$Nc=\cot\phi(Nq-1)$ [1.4]

$Nq=e^{2(3\pi/4-\phi/2)\tan\phi} / [2\cos^2(45+\phi/2)]$ [1.5]

$N\gamma=(1/2)\tan\phi(Kpr/\cos^2\phi-1)$ [1.6]

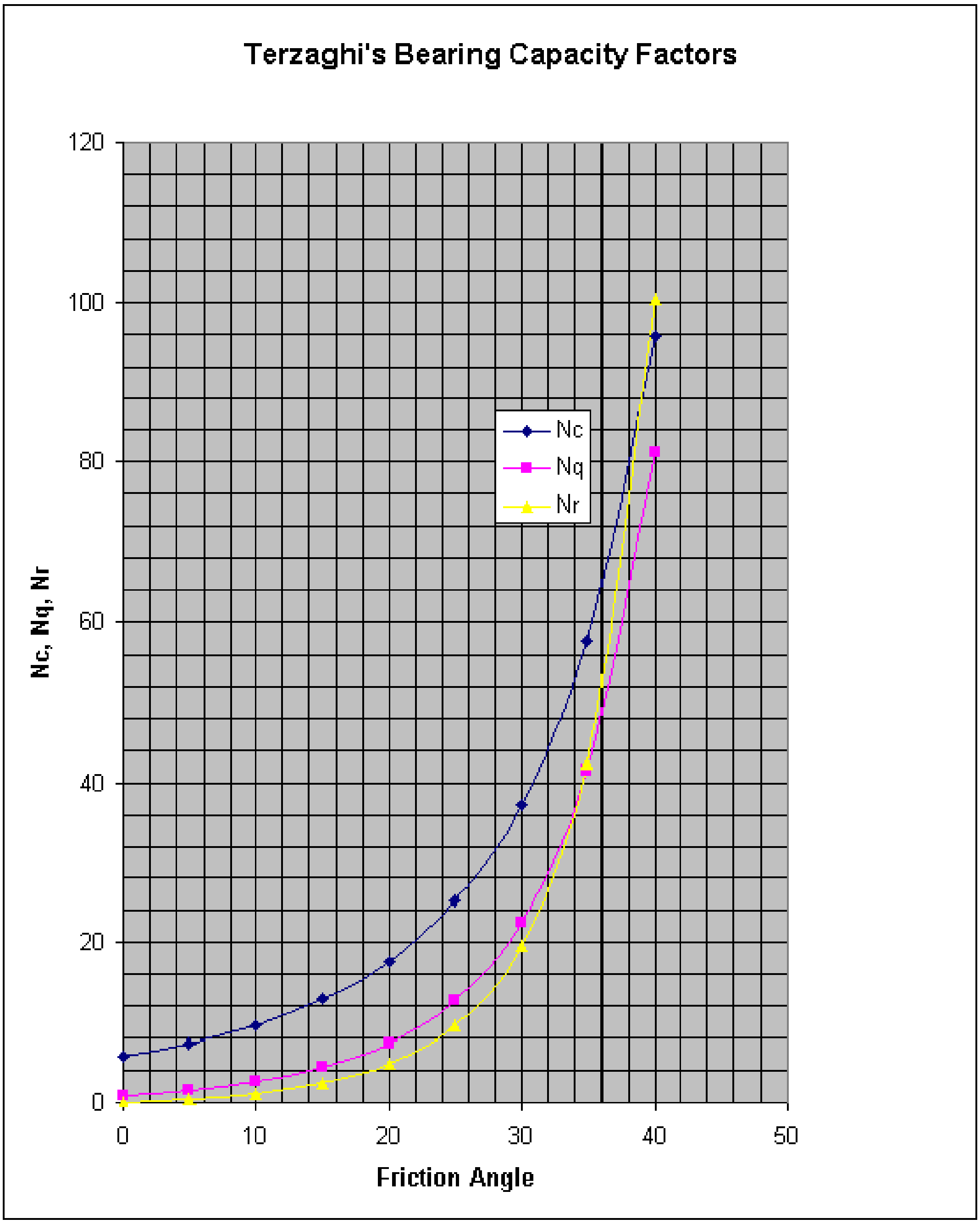
K_{pr}=passive pressure coefficient.

(Note: from Boweles, Foundation analysis and design, "Terzaghi never explained..how he obtained Kpr used to compute $N\gamma$")

Table 1: Terzaghi's Bearing Capacity Factors

ϕ	Nc	Nq	Nr
0	5.7	1	0
5	7.3	1.6	0.5
10	9.6	2.7	1.2
15	12.9	4.4	2.5
20	17.7	7.4	5
25	25.1	12.7	9.7
30	37.2	22.5	19.7
35	57.8	41.4	42.4
40	95.7	81.3	100.4

Figure 2 Terzaghi's bearing capacity factors



Example 1: Strip footing on cohesionless soil

<u>Given</u>:

- Soil properties:
- Soil type: cohesionless soil.

- Cohesion: 0 (neglectable)
- Friction Angle: 30 degree
- Unit weight of soil: 100 lbs/ft^3
- Expected footing dimensions:
- 3 ft wide strip footing, bottom of footing at 2 ft below ground level
- Factor of safety: 3

Requirement: Determine allowable soil bearing capacity using Terzaghi's equation.

Solution:

From Table 1 or Figure 1, Nc = 37.2, Nq = 22.5, Nr = 19.7 for ϕ = 30 degree
Determine ultimate soil bearing capacity using Terzaghi's bearing capacity equation for strip footing
Qu = c Nc + γ D Nq + 0.5 γ B Nγ
= 0 +100*2*22.5+0.5*100*6*19.7
= 10410 lbs/ft^2
Allowable soil bearing capacity,
Qa = Qu / F.S. = 10410 / 3 = 3470 lbs/ft^2 $\cong$ 3500 lbs/ft^2

Example 2: Square footing on clay soil

Given:

- Soil type: Clay
- Soil properties:
- Cohesion:2000 lbs/ft^2
- Friction Angle: 0 (neglectable)
- Unit weight of soil: 120 lbs/ft^3
- Expected footing dimensions:
- 6 ft by 6 ft square footing, bottom of footing at 2 ft below ground level
- Factor of safety: 3

Requirement: Determine allowable soil bearing capacity using Terzaghi's equation.

Solution:

From Table 1 or Figure 1, Nc = 5.7, Nq = 1.0, Nr = 0 for ϕ = 0 degree
Determine ultimate soil bearing capacity using Terzaghi's bearing capacity equation for square footing
Qu = 1.3 c Nc + γ D Nq + 0.4 γ B Nγ
= 1.3*1000*5.7 +120*2*1+ 0
= 7650 lbs/ft^2
Allowable soil bearing capacity,
Qa = Qu / F.S. = 7650 / 3 = 2550 lbs/ft^2 $\cong$ 2500 lbs/ft^2

Example 3: Circular footing on sandy clay

<u>Given</u>:

- Soil properties:
- Soil type: sandy clay
- Cohesion: 500 lbs/ft^2
- Friction Angle: 25 degree
- Unit weight of soil: 100 lbs/ft^3
- Expected footing dimensions:
- 10 ft diameter circular footing for a circular tank, bottom of footing at 2 ft below ground level
- Factor of safety: 3

<u>Requirement</u>:

Determine allowable soil bearing capacity using Terzaghi's equation.

<u>Solution</u>:

From Table 1 or Figure 1, Nc = 17.7, Nq = 7.4, Nr = 5.0 for ϕ = 20 degree
Determine ultimate soil bearing capacity using Terzaghi's bearing capacity equation for circular footing
$Qu = 1.3\ c\ Nc + \gamma\ D\ Nq + 0.3\ \gamma\ B\ N\gamma$
= 1.3*500*17.7 +100*2*7.4+0.3*100*10*5.0
= 17985 lbs/ft^2
Allowable soil bearing capacity,

Qa = Qu / F.S. = 17985/ 3 = 5995 lbs/ft^2 $\cong$ 6000 lbs/ft^2

Meyerhof's general bearing capacity equations

- Bearing capacity equation for vertical load, inclined load
- Meyerhof's bearing capacity factors
- Chart for Bearing capacity factor
- Example 4: Strip footing on clayey sand
- Example 5: Rectangular footing on sandy clay
- Example 6: Square footing with incline loads

Meyerhof’s general bearing capacity equations

Vertical load:

$Qu = c\ Nc\ Sc\ Dc + \gamma\ D\ Nq\ Sq\ Dq + 0.5\ \gamma\ B\ N\gamma\ S\gamma\ D\gamma$ [1.7]

Inclined load:

$Qu = c\ Nc\ Sc\ Dc\ Ic + \gamma\ D\ Nq\ Sq\ Dq\ Iq + 0.5\ \gamma\ B\ N\gamma\ S\gamma\ D\gamma\ I\gamma$ [1.8]

Where:

Nc, Nq, Nr: Meyerhof’s bearing capacity factors depend on soil friction angle, ϕ.

$Nc = \cot\phi\ (Nq - 1)$ [1.9]

$Nq = e^{\pi\tan\phi}\ \tan^2(45+\phi/2)]$ [1.10]

$N\gamma = (Nq-1)\tan(1.4\phi)$ [1.11]

Sc, Sq, $S\gamma$: shape factors

Dc, Dq, $D\gamma$: depth factors

Ic, Iq, $I\gamma$: incline load factors

Friction angle	Shape factor	Depth factor	Incline load factors
Any ϕ	$Sc=1+0.2Kp(B/L)$	$Dc=1+0.2\sqrt{Kp}\ (B/L)$	$Ic=Iq=(1-\theta/90°)^2$
$\phi = 0$	$Sq=S\gamma=1$	$Dq=D\gamma=1$	$I\gamma=1$
$\geq\phi 10°$	$Sq=S\gamma=1+0.1Kp(B/L)$	$Dq=Dr=1+0.1\sqrt{Kp}\ (D/B)$	$I\gamma=(1-\theta/\phi)^2$

C: Cohesion of soil

γ : unit weight of soil

D: depth of footing

B, L: width and length of footing

$Kpr = \tan^2(45+\phi/2)$, passive pressure coefficient.

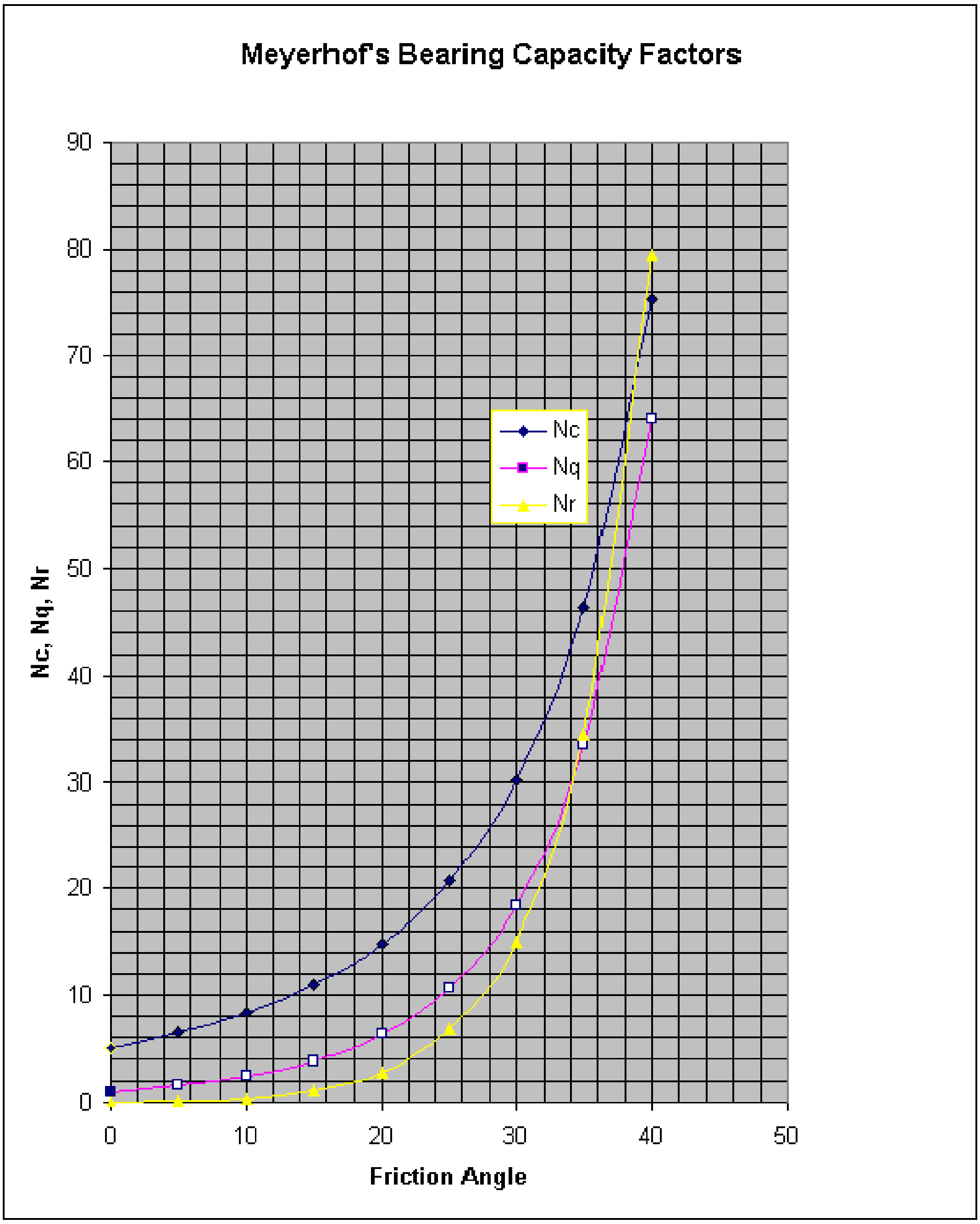
θ = angle of axial load to vertical axis

Table 2: Meyerhof’s bearing capacity factors

ϕ	Nc	Nq	Nr
0	5.1	1	0
5	6.5	1.6	0.1
10	8.3	2.5	0.4
15	11	3.9	1.2
20	14.9	6.4	2.9
25	20.7	10.7	6.8
30	30.1	18.4	15.1
35	46.4	33.5	34.4

40	75.3	64.1	79.4

Figure 2: Meyerhof's bearing capacity factors



Example 4: Strip footing on clayey sand

<u>Given</u>:

- Soil properties:
- Soil type: clayey sand.
- Cohesion: 500 lbs/ft^2
- Cohesion: 25 degree
- Friction Angle: 30 degree
- Unit weight of soil: 100 lbs/ft^3
- Expected footing dimensions:
- 3 ft wide strip footing, bottom of footing at 2 ft below ground level
- Factor of safety: 3

Requirement:

Determine allowable soil bearing capacity using Meyerhof's equation.

Solution:

Determine ultimate soil bearing capacity using Meyerhof's bearing capacity equation for vertical load.
Passive pressure coefficient
$Kpr = \tan^2(45+\phi/2) = \tan^2(45+25/2) = 2.5$
Shape factors:
$Sc=1+0.2Kp(B/L) = 1+0.2*2.5*(0)=1$
$Sq=S\gamma=1+0.1Kp(B/L) = 1+0.1*2.5*(0) = 1$
Depth factors:
$Dc=1+0.2\sqrt{Kp}\ (B/L) = 1+0.2*\sqrt{2}\ (0) = 1$
$Dq=D\gamma=1+0.1\sqrt{Kp}\ (D/B) = 1+0.1*\sqrt{2.5}\ (3/3) = 1.16$
From Table 2 or Figure 2, Nc = 20.7, Nq = 10.7, Nr = 6.8 for ϕ = 25 degree
$Qu = c\ Nc\ Sc\ Dc + \gamma\ D\ Nq\ Sq\ Dq + 0.5\ \gamma\ B\ N\gamma\ S\gamma\ D\gamma$
$= 500*20.7*1*1 +100*3*10.7*1*1.16+0.5*100*3*6.8*1*1.16$
$= 15257\ lbs/ft^2$
Allowable soil bearing capacity,
$Qa = Qu / F.S. = 15257 / 3 = 5085\ lbs/ft^2 \cong 5000\ lbs/ft^2$

Example 5: Rectangular footing on sandy clay

Given:

- Soil properties:
- Soil type: sandy clay
- Cohesion: 500 lbs/ft^2
- Friction Angle: 20 degree
- Unit weight of soil: 100 lbs/ft^3
- Expected footing dimensions:
- 8 ft by 4 ft rectangular footing, bottom of footing at 3 ft below ground level.
- Factor of safety: 3

<u>Requirement</u>:

Determine allowable soil bearing capacity using Meyerhof's equation.

<u>Solution</u>:

Determine ultimate soil bearing capacity using Meyerhof's bearing capacity equation for vertical load.
Passive pressure coefficient
$Kpr = \tan^2(45+\phi/2) = \tan^2(45+20/2) = 2.$
Shape factors:
$Sc=1+0.2Kp(B/L) = 1+0.2*2*(4/8)=1.2$
$Sq=S\gamma=1+0.1Kp(B/L) = 1+0.1*2*(4/8) = 1.1$
Depth factors:
$Dc=1+0.2\sqrt{Kp}\ (B/L) = 1+0.2*\sqrt{2}\ (4/8) = 1.14$
$Dq=D\gamma=1+0.1\sqrt{Kp}\ (D/B) = 1+0.1*\sqrt{2}\ (3/4) = 1.1$
From Table 2 or Figure 2, Nc = 14.9, Nq = 6.4, Nr = 2.9 for ϕ = 20 degree
$Qu = c\ Nc\ Sc\ Dc + \gamma\ D\ Nq\ Sq\ Dq + 0.5\ \gamma\ B\ N\gamma\ S\gamma\ D\gamma$
$= 500*14.9*1.2*1.14 +100*3*6.4*1.1*1.1+0.5*100*4*2.9*1.1*1.1$
$= 13217$ lbs/ft^2
Allowable soil bearing capacity,
$Qa = Qu / F.S. = 13217 / 3 = 4406$ lbs/ft^2 $\cong$ 4400 lbs/ft^2

Example 6: Square footing with incline loads

<u>Given</u>:

- Soil properties:
- Soil type: sandy clay
- Cohesion: 1000 lbs/ft^2
- Friction Angle: 15 degree
- Unit weight of soil: 100 lbs/ft^3
- Expected footing dimensions:
- 8 ft by 8 ft square footing, bottom of footing at 3 ft below ground level.
- Expected column vertical load = 100 kips
- Expected column horizontal load = 20 kips
- Factor of safety: 3

<u>Requirement</u>:

Determine allowable soil bearing capacity using Meyerhof's equation.

<u>Solution</u>:

Determine ultimate soil bearing capacity using Meyerhof's bearing capacity equation for vertical load.
Passive pressure coefficient

$Kpr = \tan^2(45+\phi/2) = \tan^2(45+15/2) = 1.7$
Shape factors:
 $Sc=1+0.2Kp(B/L) = 1+0.2*1.7*(8/8)=1.34$
 $Sq=S\gamma=1+0.1Kp(B/L) = 1+0.1*1.7*(8/8) =1.17$
Depth factors:
 $Dc=1+0.2\sqrt{Kp} (B/L) = 1+0.2*\sqrt{1.7} (8/8) = 1.26$
 $Dq=D\gamma=1+0.1\sqrt{Kp} (D/B) = 1+0.1*\sqrt{1.7} (3/8) = 1.05$
Incline load factors:
$\theta = \tan^{-1} (20/100) = 11.3°$

$Ic=Iq=(1-\theta/90°)^2=(1-11.3/90)^2= 0.76$
$I\gamma=(1-\theta/\phi)^2=(1-11.3/15)^2=0.06$

From Table 2 or Figure 2, Nc = 11, Nq = 3.9, Nr = 1.2 for ϕ = 15 degree
$Qu = c\ Nc\ Sc\ Dc\ Ic + \gamma\ D\ Nq\ Sq\ Dq\ Iq + 0.5\ \gamma\ B\ N\gamma\ S\gamma\ D\gamma\ I\gamma$
$= 500*11*1.34*1.26*0.76+100*3*3.9*1.17*1.05*0.76+0.5*100*8*1.17*1.05*0.06$
$= 8179$ lbs/ft^2
Allowable soil bearing capacity,
Qa = Qu / F.S. = 8179 / 3 = 2726 lbs/ft^2 $\cong$ 2700 lbs/ft^2

Bearing capacity from SPT numbers

One of most commonly method for determining allowable soil bearing capacity is from standard penetration test (SPT) numbers. It is simply because SPT numbers are readily available from soil boring. The equations that are commonly used were proposed by Meryerhof based on one inches of foundation settlement. Bowles revised Meyerhof's equations because he believed that Meryerhof's equation might be conservative.

Meryerhof's equations:

For footing width, 4 feet or less:
$Qa = (N/4) / K$ [1.12]
For footing width, greater than 4 ft:
$Qa = (N/6)[(B+1)/B]^2 / K$ [1.13]

Bowles' equations:

For footing width, 4 feet or less:
$Qa = (N/2.5) / K$ [1.14]
For footing width, greater than 4 ft:
$Qa = (N/4)[(B+1)/B]^2 / K$ [1.15]
Qa: Allowable soil bearing capacity, in kips/ft2.
N: SPT numbers below the footing.

B: Footing width, in feet
K = 1 + 0.33(D/B) ≤ 1.33
D: Depth from ground level to the bottom of footing, in feet.

Example 7: Determine soil bearing capacity by SPT numbers
<u>Given</u>

- Soil SPT number: 10
- Footing type: 3 feet wide strip footing, bottom of footing at 2 ft below ground surface.

<u>Requirement</u>: Estimate allowable soil bearing capacity based on.
<u>Solution:</u>
Meryerhof's equation
K = 1+0.33(D/B) = 1+0.33*(2/3) = 1.22
Qa = (N/4) / K = (10 /4) /1.22 = 2 kips/ft2
Bowles' equation:
Qa = (N/2.5) / K = (10 /2.5) /1.22 = 3.3 kips/ft2

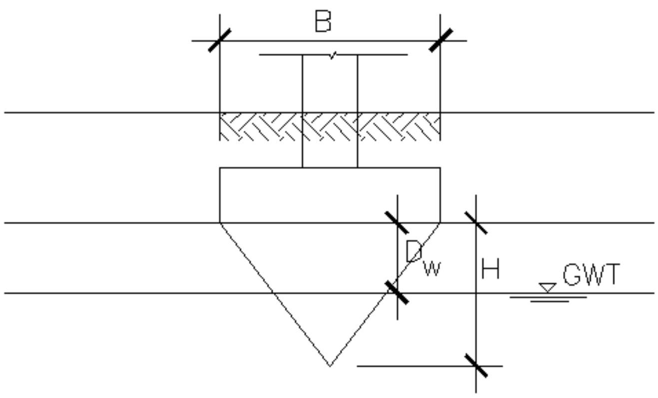
Example 8: Determine soil bearing capacity by SPT numbers

<u>Given:</u>

- Soil SPT number: 20
- Footing type: 8 feet wide square footing, bottom of footing at 4 ft below ground surface.

<u>Requirement</u>: Estimate allowable soil bearing capacity based on Meryerhof's equation.
<u>Solution:</u>
Meryerhof's equation
K = 1+0.33(D/B) = 1+0.33*(4/8) = 1.17
Qa = (N/6)[(B+1)/B]2 / K = (20/6)[(8+1)/8]2 /1.17 = 3.6 kips/ft2
Bowles' equation:
Qa = (N/4)[(B+1)/B]2 / K = (20/4)[(8+1)/8]2 /1.17 = 5.4 kips/ft2

. Effect of water table on soil bearing capacity



When the water table is above the wedge zone, the soil parameters used in the bearing capacity equation should be adjusted. Bowles proposed an equation to adjust unit weight of soil as follows:

$$\gamma_e=(2H-D_w)(D_w/H^2)\gamma_m+(\gamma'/H^2)(H-D_w)^2 \qquad [1.16]$$

Where

- γ_e = Equivalent unit weight to be used in bearing capacity equation,
- H = 0.5Btan(45+ϕ/2), is the depth of influence zone,
- D_w= Depth from bottom of footing to ground water table,
- γ_m = Moist unit weight of soil above ground water table,
- γ' = Effective unit weight of soil below ground water table.

Conservatively, one may use the effective unit water under ground water table for calculation. Equation 1.16 can also used to adjust cohesion and friction angle if they are substantially differences.

Example 9: Determine equivalent unit weight of soil to calculate soil bearing capacity with the effect of ground water table

<u>Given:</u>

- Moist unit weight of soil above ground water table: 120 lb/ft^3.
- Moist content = 20%
- Friction angle, ϕ = 25 degree
- Cohesion of soil above ground water table: 1000 lb/ft^2.
- Cohesion of soil below ground water table: 500 lb/ft^2.
- Footing: 8 feet wide square footing, bottom of footing at 2 ft below ground surface.
- Location of ground water table: 6 ft below ground water surface.

<u>Requirement</u>: Determine equivalent unit weight of soil to be used for calculating soil bearing capacity.

<u>Solution:</u>

Determine equivalent unit weight:

Dry unit weight of soil, $\gamma_{dry} = \gamma_m /(1+ \omega) = 120/(1+0.2) = 100$ lb/ft^3.

Volume of solid for 1 ft^3 of soil, $V_s = \gamma_{dry} / (G_s\gamma_w) = 100 / (2.65*62.4) = 0.6$ ft^3.

Volume of void for 1 ft^3 of soil, $V_v = 1-V_s=1-0.6=0.4$ ft^3.

Saturate unit weight of soil, $\gamma_{sat} = \gamma_{dry} + \gamma_w V_v = 100+62.4*0.4=125$ ft^3.

Effective unit weight of soil = $\gamma_{sat} - \gamma_w = 125-62.4=62.6$ ft^3.

Effective depth, H = 0.5B tan(45+ϕ/2) = 0.5*8*tan (45+30/2) = 6.9 ft

Depth of ground water below bottom of footing, D_w= 6-2 = 4 ft

Equivalent unit weight of soil,

$$\gamma_e = (2H-D_w)(D_w/H^2)\gamma_m+(\gamma'/H^2)(H-D_w)^2$$
$$=(2*6.9-4)(4/6.9^2)*100+(62.6/6.9^2)(6.9-4)^2$$

$= 93.4\ \text{lb/ft}^3$.

ASCE 7-98 SEISMIC LOAD CALCULATION

- **Contents:**

- ASCE 7-98 Equivalent lateral force procedure
- Example 1: Bearing wall systems with ordinary reinforced masonry shear wall
- Example 2: Building frame systems with ordinary steel concentric braced frame
- Example 3: Building frame systems with ordinary reinforced concrete shear wall

ASCE 7-98 Equivalent lateral force procedure

1. Determine weight of building, W.
2. Determine 0.2 second response spectral acceleration, S_S from Figure 9.4.1.1 (a) or (c), (e), (f), (g-1), (h-1), (i), and (j)
3. Determine 1 second response spectral acceleration, S_1 from Figure 9.4.1.1 (b), or (d), (f), (g-2), (h-2), (i) and (j)
4. Determine Site class from Table 9.4.1.2
5. Determine site coefficient, F_a, from Table 9.4.1.2.4a.
6. Determine site coefficient, F_v, from Table 9.4.1.2.4b
7. Determine adjusted maximum considered earthquake spectral response acceleration parameters for short period, S_{MS} and at 1 second period, S_{M1}.

$S_{MS} = F_a S_s$ (Eq. 9.4.1.2.4-1)

$S_{M1} = F_v S_1$ (Eq. 9.4.1.2.4-2)

8. Determine deign spectral response acceleration parameters for short period, S_{DS} and at 1 second period, S_{D1}.

$S_{DS} = (2/3)\, S_{MS}$ (Eq. 9.4.1.2.5-1)

$S_{D1} = (2/3)\, S_{M1}$ (Eq. 9.4.1.2.5-2)

9. Determine Important factor, I, from Table 9.1.4,
10. Determine Seismic design category from Table 9.4.2.1
11. Determine Response modification factor, R. from Table 9.5.2.2 and check building height limitation
12. Determine seismic response coefficient from Eq. 9.5.3.2.1-1

$C_s = S_{DS} / (R/I)$

13. Determine approximate fundamental period from Eq. 9.5.3.3-1

$T = C_T\, h_n^{0.75}$

where

h_n is the height of building above base.

C_T is building period coefficient, 0.035 for moment resisting frame of steel, 0.03 for moment resisting frame of concrete and eccentrically braced steel frame 0.02 for all other building.

14. Determine Maximum seismic response coefficient Eq. 9.3.2.1-2

$C_{smax} = S_{D1} / [(R/I)\ T]$

15. Determine minimum seismic response coefficient from Eq. 9.5.3.2.1-3

$C_{smin} = 0.044\ S_{DS} I$

16. If it is design category E or F, or S1 is equal or grater than 0.6g, calculate minimum seismic response coefficient from Eq. 9.5.3.2.1-4

$C_{smin} = 0.5\ S_1 / (R/I)$

17. Determine Seismic response coefficient based on result of steps 12 to 16 and calculate
seismic base shear from Eq. 9.3.2-1. for strength design or load and resistance factor design.

$V = C_s\ W$

18. For service load design, multiply the seismic base shear by 0.7

$V_s = 0.7\ V$

Example 1: Bearing wall systems with ordinary reinforced masonry shear wall

Given:

Code: ASCE 7-98 Equivalent lateral force procedure
Design information:
Weight of building, W=500 kips
0.2 second response spectral acceleration, $S_S = 0.25$
1 second response spectral acceleration, $S_1 = 0.1$
Soil profile class: E
Bearing wall systems with ordinary reinforced masonry shear wall
Building category I

Requirement: Determine seismic base shear

Solution;

Site coefficient, $F_a = 2.5$

Site coefficient, $F_v = 3.5$

Design spectral response acceleration parameters

$S_{MS} = F_a\ S_s = 0.625$ (Eq. 9.4.1.2.4-1)
$S_{M1} = F_v\ S_1 = 0.35$ (Eq. 9.4.1.2.4-2)
$S_{DS} = (2/3)\ S_{MS} = 0.42$ (Eq. 9.4.1.2.5-1)
$S_{D1} = (2/3)\ S_{M1} = 0.233$ (Eq. 9.4.1.2.5-2)

Seismic design category C from Table 9.4.2.1a, category D based on Table 9.4.2.1b. Use category D.

From Table 9.5.2.2, Bearing wall system with ordinary reinforced masonry shear wall is NP (not permitted). Need to change structural system.

Example 2: Building frame systems with ordinary steel concentric braced frame

Given:

Code: ASCE 7-98 Equivalent lateral force procedure
Design information:
Weight of building, W = 500 kips
0.2 second response spectral acceleration, $S_s = 0.25$
1 second response spectral acceleration, $S_1 = 0.1$
Building frame systems with ordinary steel concentric braced frame
Building category I
Building height: 30 ft

Requirement: Determine seismic base shear

Solution:

Site coefficient, $F_a = 2.5$
Site coefficient, $F_v = 3.5$

Design spectral response acceleration parameters

$S_{MS} = F_a S_s = 0.625$ (Eq. 9.4.1.2.4-1)
$S_{M1} = F_v S_1 = 0.35$ (Eq. 9.4.1.2.4-2)
$S_{DS} = (2/3) S_{MS} = 0.42$ (Eq. 9.4.1.2.5-1)
$S_{D1} = (2/3) S_{M1} = 0.233$ (Eq. 9.4.1.2.5-2)

Seismic design category C from Table 9.4.2.1a, category D based on Table 9.4.2.1b.
Use category D.
From Table 9.5.2.2, Building frame system with ordinary steel concentric braced frame is 160 ft
Response modification factor, R = 5
Important factor, I = 1 (Table 9.1.4)
Seismic response coefficient (Eq. 9.5.3.2.1-1)

$C_s = S_{DS} / (R/I) = 0.083$

Fundamental period (Eq. 9.5.3.3.1)

$T = C_T h_n^{0.75} = 0.0256$

Maximum seismic response coefficient (Eq. 9.5.3.2.1-2)

$C_{smax} = S_{D1} / [(R/I) T] = 0.182$

Minimum seismic response coefficient (Eq. 9.5.3.2.1-3)

$C_{smin} = 0.044 S_{DS} I = 0.018$

Seismic base shear

$V = C_s W = 42$ kips

Seismic base shear in service load,

$V_s = 0.7 V = 29$ kips

Example 3: Building frame systems with ordinary reinforced concrete shear wall

Given:

Code: ASCE 7-98 Equivalent lateral force procedure
Design information:
Weight of building, W = 1000 kips
0.2 second response spectral acceleration, Ss = 0.5
1 second response spectral acceleration, $S_1 = 0.15$
Soil profile class: C
Building frame systems with ordinary reinforced concrete shear wall
Building category II
Building height: 40 ft

Requirement: Determine seismic base shear

Solution:

Site coefficient, $F_a = 1.2$
Site coefficient, $F_v = 1.65$
Design spectral response acceleration parameters

$S_{MS} = F_a S_s = 0.6$ (Eq. 9.4.1.2.4-1)
$S_{M1} = F_v S_1 = 0.25$ (Eq. 9.4.1.2.4-2)
$S_{DS} = (2/3)\ S_{MS} = 0.4$ (Eq. 9.4.1.2.5-1)
$S_{D1} = (2/3)\ S_{M1} = 0.17$ (Eq. 9.4.1.2.5-2)

Seismic design category C from Table 9.4.2.1a, category C based on Table 9.4.2.1b. Use category C.
From Table 9.5.2.2, Building frame system with ordinary steel concentric braced frame is 160 ft
Response modification factor, R = 5
Important factor, I = 1 (Table 9.1.4)
Seismic response coefficient (Eq. 9.5.3.2.1-1)

$C_s = S_{DS} / (R/I) = 0.1$

Fundamental period (Eq. 9.5.3.3.1)

$T = C_T\ h_n^{0.75} = 0.318$

Maximum seismic response coefficient (Eq. 9.5.3.2.1-2)

$C_{smax} = S_{D1} / [(R/I)\ T] = 0.104$

Minimum seismic response coefficient (Eq. 9.5.3.2.1-3)

$C_{smin} = 0.044\ S_{DS} I = 0.018$

Seismic base shear

$V = C_s W = 100$ kips

Seismic base shear in service load,

$V_s = 0.7\ V = 70$ kips

Design of Unreinforced Masonry Wall

General:

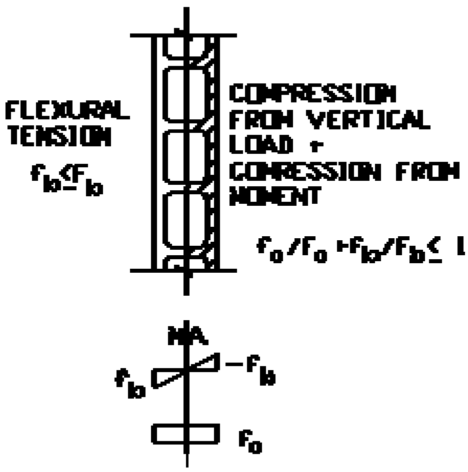
Unreinforced masonry walls are often used as load bearing or non-loading interior wall in one story building. Although it is called “unreinforced”, the masonry wall still needs to be reinforced with joint reinforcements. In addition, ordinary and detailed plan masonry walls are allowed as shear walls in seismic design category A & B. But it still needs to meet code required minimum reinforcement requirements. By definition of ACI 530 Section 1.6, the tensile strength of masonry is considered, but the strength of reinforcing steel is neglected.

Load on masonry walls:

1. Vertical dead load, live load, snow load, etc.
2. Lateral load from wind, seismic, earth pressure etc.

Stresses in concrete masonry wall:

1. Compressive stress from vertical load
2. Compressive stress from flexural moment due to lateral load.
3. Tensile stress from flexural moment due to lateral load, eccentric moment, etc.



Design requirements:

When the wall, pilaster, and column is subjected to axial compression and flexure.

1. The maximum compression stress shall satisfy the following equation

   $f_a/F_a + f_b/F_b \leq 1$ (ACI 530 Eq. 2-10)

Where, f_a is compressive stress from axial load, f_b is compressive stress from flexure; F_a and F_b, F_v are allowable compressive stress and tensile stress calculation from equation below:

For member with $h/r \le 99$:

$$F_a = (1/4)\, f_m' \{1-[h/(140r)]^2\} \qquad \text{(ACI 530 2-12)}$$

For member with $h/r > 99$:

$$F_a = (1/4)\, f_m' (70r/h)^2 \qquad \text{(ACI 530 2-13)}$$

Where h is effective height of wall, column or pilaster, r is radius of gyration, fm' is compressive strength of masonry

Allowable compressive stress from flexure:

$$Fb = (1/3)\, f_m' \qquad \text{(ACI 530 2-14)}$$

2. The maximum axial force P shall satisfy the following equation

$$P \le (1/4)\, P_e \qquad \text{(ACI 530 Eq. 2-11)}$$

Where Pe is calculated as

$$P_e = [\pi^2 E_m I/h^2][1-0.577*e/r)^2 \qquad \text{(ACI 530 2-15)}$$

Where E_m is elastic modulus of masonry, I is moment of inertia, h is the height of wall, column or plaster, e is eccentricity of axial load, r is radius of gyration.

3. The tensile stress due to flexure shall not exceed the value listed in ACI 530 Table 2.2.3.2. as shown below:

Allowable flexural tension stress for hollow core concrete masonry unit, psi.

Masonry Mortar

Portland cement/lime or mortar cement Masonry cement or air entrained Portland cement/lime

Normal to bed joints

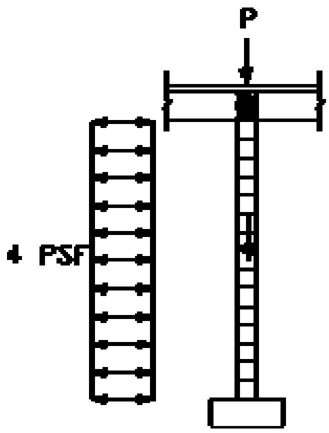
Ungrouted hollow units 25 19 15 9

Grout hollow units 68 58 41 29

Parallel to bed joints in running bond

Ungrouted or partially grout hollow units	50	38	30	19
Fully grouted hollow units	80	60	48	30

Example 1: Design of an interior unreinforced load bearing masonry wall



Design data:
Roof dead load: 20 psf
Roof live load: 20 psf
Tributary width: 30 ft
Height of wall: 12 ft
Normal width of wall: 8 in
Assume minimum eccentricity: 0.8 in
Seismic load from ASCE 7: 4 psf
Requirement: Check if an 8 in unreinforced masonry wall is adequate
Solution:
Axial load per foot width of wall from roof, P = (20 psf+20 psf)*30 psf = 1200 lb/ft
Eccentricity: e = 0.8 in
Eccentric moment: M_c = 1200*0.8/12 = 80 lb/ft
Seismic moment: M_s = 5 psf * $(12ft)^2/8$ = 90 lb-ft/ft
Moment per foot width of wall, M = (80 + 90) lb-ft/ft = 170 lb-ft/ft
Width of wall: 7.625 in
Cross section area: A = 42.8 in^2.
Moment of inertia: I = 330.9 in^4.
Section modulus: S = 86.8 in^3.
Radius of gyration: r = 2.78 in
Check flexural tensile stress: fb = M/S = 170*12/86.8 = 23.5 psf
Less than allowable tensile stress 25 psi for ungrouted hollow unit O.K.
Check compressive stress:
Weight of wall at mid-height: W = 55 psf * 6 ft = 330 lb/ft
Axial compressive stress: f_a = (P+W)/A = 35.8 psi/ft
Slenderness ratio: h/r = 12*12/2.78 = 51.7 < 99
Use 1900 psi concrete masonry units with type S mortar,
Compressive strength of concrete masonry, f_m' = 1500 psi
$F_a = (1/4) f_m' \{1-[h/(140r)]^2\} = 323.7$ psi

Allowable flexural strength:

$Fb = (1/3)\ f_m' = 500$ psi

Combined stress equation:

$f_a/F_a + f_b/F_b = 0.158 < 1$ O.K.

Check axial force:

Elastic modulus, Em = 900 fm' = 1.35×10^6 psi

$P_e = [\pi^2 E_m I/h^2][1-0.577*e/r)^2 = 1.23 \times 10^5$ lb

$(1/4)\ P_e = 3.09 \times 10^4$ lb > 1200+330 = 1530 lb O.K.

Concrete Masonry Design

Mechanical properties of concrete masonry:

Topics:

- Properties of concrete masonry units
- Mortar
- Grout
- Reinforcement
- Bond types
- Compressive strength of concrete masonry, f_m'
- Modulus of elasticity of concrete masonry
- Masonry control joints

General: The mechanical properties of concrete masonry wall, pilaster, column, lintel etc. depends on the properties of concrete masonry units, mortar, grout, reinforcement and how the units were arranged.

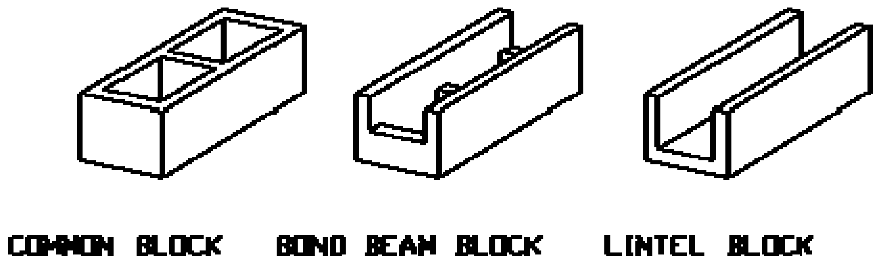
Properties of concrete masonry units:

Material: Portland cement, Hydrated lime, Pozzolans, Normal weight or light weight aggregates.

Dimension of commonly used Concrete masonry units:

Nominal size	Actual size	Face Wall thickness	Cross section (in^2/ft)	Section modulus (in^4/ft)	Moment of inertia (in^3/ft)	Radius of gyration (in)
8x4x16	7-5/8"x3-5/8"x15-5/8"	3/4"	22.4	21.2	38.5	1.31
8x8x16	7-5/8"x7-5/8"x15-5/8"	1-1/4"	42.8	86.8	330.9	2.78
8x12x16	7-5/8"x12-5/8"x15-5/8"	1-1/2"	57	273.6	1043	4.28

Type of commonly used hollow core concrete blocks :



Mortar:

Material: Portland cement/Masonry cement water, lime, sand, and admixtures

Mixes: Type M, S, N and O. depend on proportion of mixes. Compressive strength of cues for mortar types are as follows:

Mortar Types	Average compressive strength at 28 days (psi)
M	2500
S	1800
N	750
O	350

Grout:

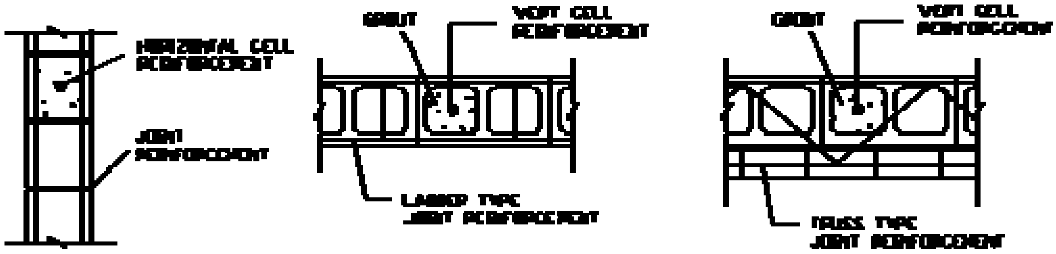
Ingredients: Portland cement, fine aggregates, coarse aggregate, Lime.

Mix proportions: depending on its strength, a commonly use mix is one part of cement with 1/10 of lime, three parts of fine aggregates, and 2 parts of coarse aggregate with maximum aggregate size limits by the grout space.

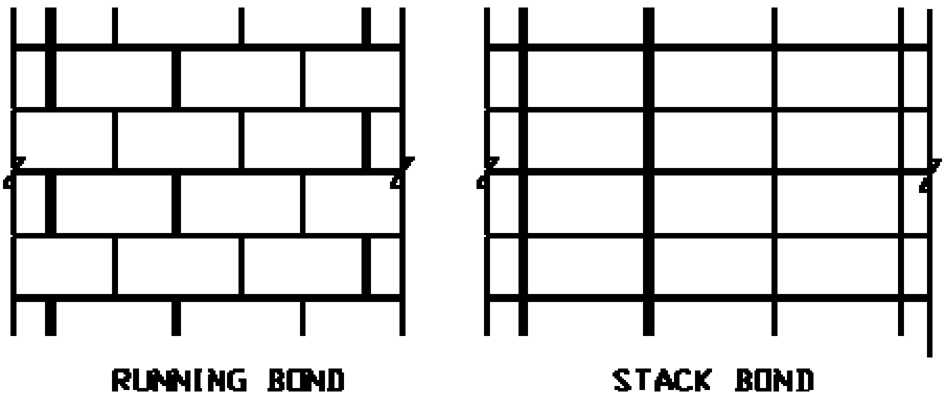
Reinforcement:

Joint reinforcement: Ladder type or truss type, usually, 9 or 10-gage wires.

Cell reinforcement: rebars same as concrete reinforcement.



<u>Bond types</u>: Running bond and stack bond.



<u>Compressive strength of concrete masonry, f_m’</u>

The compressive strength of masonry varies with the type of mortar and the strength of units. There are two methods to determine the strength of masonry during construction. One is based on prism test; the other is based on values specified in the codes. The values list in International Building Code, 2003, Table 2105.2.2.1.2 are as follows:

Net area compressive strength of concrete masonry units (psi) Net area compressive strength of masonry

f_m’(psi)	Type M or S mortar	Type N mortar
1250	1300	1000
1900	2150	1500
2800	3050	2000
3750	4050	2500
4800	5250	3000

Modulus of elasticity of concrete masonry:

$E_m = 900\ f_m'$ (ACI 530 Section 1.8.2.2.1)

Thermal expanson coefficients: $k_t = 4.5 \times 10^{-6}$ in/in/^oF

Shrinkage coefficient:

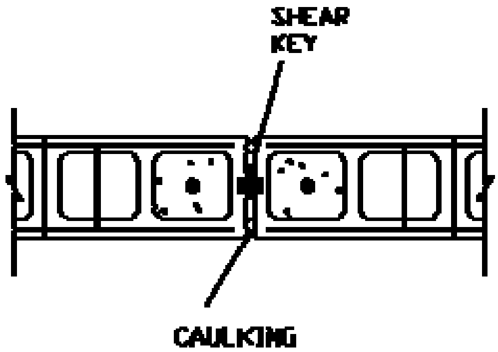
Masonry made of non-moisture controlled concrete masonry units: $k_m = 0.5s_1$

Masonry made of moisture controlled concrete masonry units: $k_m = 0.15s_1$

Where $s_1 = 6.5\text{x}10^{-6}$ in/in.

Masonry control joints:

Masonry control joints are used to allow expansion and shrinkage of masonry wall, minimize random cracks, and distress. Spacing of masonry control joints recommended in the commentary of ACI 530.1 is 25 ft or 3 times of wall height. It also recommends placing control joints at returns and jambs of openings. Shear key may be used to control wall movement in the out-of-plan direction.



MASONRY CONTROL JOINT

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